Image Regularization using Neural Networks

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Background

Image restoration is a field which utilises the tools of linear algebra and functional analysis, often by means of *regularization techniques* [1]. Traditionally a blurred image $B(s) \in \mathcal{Y}$ is modelled as the convolution of the unknown, clear image function $I(x) \in \mathcal{X}$ with some blurring kernel,

$$B(s) = \int_\Omega I(x)k(x,s)dx, \tag{1}$$

where $\mathcal{X}$ and $\mathcal{Y}$ are taken to be Hilbert spaces. The blurred image is never known exactly, as it will be polluted with measurement errors, often assumed to be white noise. Instead we have $\widetilde{B}(s) = B(s) + \eta(s)$, $\|\eta(s)\| < \delta$. Such convolution operators induced by blurring kernels $k(x,s)$ are generally compact, and the solution to (1) is discontinuous with respect to $B(s)$. Thus inversion using $\widetilde{B}(s)$ will produce a reconstruction with unbounded errors. Instead one generally uses stabilized reconstruction techniques, recovering the original image by a de-convolution method such as Tikhonov regularisation coupled with some sort of penalty term, e.g., the L^2-norm of the reconstructed image or its total variation; see, e.g., [2] and [3].

Objectives

In this project we will explore, design, and implement an artificial neural network (ANN) as an alternative solution strategy for either all or part of this process. This is inspired by papers describing the use of ANNs for this and similar processes, such as, [4], and the guidance of our supervisor. From our preliminary research of the problem, a possible application is the use of an ANN to generate an approximation of the blur kernel used, the form of which is traditionally assumed a priori, providing a more general solution when training data exists.

We will thereafter adapt our approach to specific applications, where the easily trained nature of an ANN could allow us to more contextually interpret data, e.g., using a data-set of human faces to infer local movement when relevant. We will publish any results either in a journal and/or in a public code repository.

Tasks

The first task of the project is to familiarise ourselves with currently-used regularization techniques as discussed above, implementing standard methods using Python. This will require developing an understanding of the model computationally and analytically. Alongside this, we will develop our understanding of neural networks mathematically and practically to best apply them to the problem.

We will then conduct a theoretical analysis determining how an ANN could be used to develop a for more efficient, accurate or general reconstruction algorithm, with possibilities as discussed above.

With our designed architecture in mind, we will implement and benchmark our procedures against current algorithms, discussing the relative merit of the results. Each of these tasks will likely take a week or so, using the full allotted time for the project.

References

[1] Heinz W. Engl, Martin Hanke, and Andreas Neubauer. *Regularization of Inverse Problems.* Kluwer Academic Publishers Group, Dordrecht, 1996.

[2] Antonin Chambolle, *An Algorithm for Total Variation Minimization and Applications*, 2004.

[3] Andrew Y. Ng *Feature selection, L1 vs. L2 regularization, and rotational invariance*

[4] Jian Sun, Wenfei Cao, Zongben Xu, Jean Ponce, *Learning a Convolutional Neural Network for Non-uniform Motion Blur Removal*, 2015